Online Appendix to "A Jamming Theory of Politics"

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Lemma 1. *In equilibrium, the receiver believes that the state is w with probability 1 only if $m_1 = m_2 = w$. If Opposing Senders is satisfied, then it is possible in equilibrium for such confirmatory messages to be sufficient for the receiver to maintain such a belief.*

Proof. Battaglini (2002) proves the first claim, and Krishna and Morgan (2001) prove the second. Neither assumes privately known preferences, but this assumption does not alter their proofs. ■

Lemma 2. *If players use the strategies*

$$\begin{aligned} m_1(w, x_1) &= \left\{ \begin{array}{c} m_J(w) \\ w \end{array} \right\} \text{ if } w \in \left\{ \begin{array}{l} (y_d - 2|x_1|, y_d) \\ \text{otherwise} \end{array} \right\}, \\ m_2(w, x_2) &= \left\{ \begin{array}{c} m_J(w) \\ w \end{array} \right\} \text{ if } w \in \left\{ \begin{array}{l} (y_d, y_d + 2|x_2|) \\ \text{otherwise} \end{array} \right\}, \text{ and} \\ y(m_1, m_2) &= \left\{ \begin{array}{c} y_d \\ m_1 \end{array} \right\} \text{ if } m_1 \left\{ \begin{array}{c} \neq \\ = \end{array} \right\} m_2, \end{aligned} \tag{1}$$

then

$$y_d = wh(w|w, m_J(w)) + m_J(w)(1 - h(w|w, m_J(w))). \tag{2}$$

must hold whenever $m_1 \neq m_2$.

Proof. For every w at least one sender sends a truthful message, so the receiver knows $w \in \{m_i, m_j\}$. Since the receiver has symmetric, single-peaked preferences and type 0, she chooses $y_d = E_w(w|m_1, m_2)$. ■

Lemma 3. *Assume w and $m_J(w)$ satisfy equation (2), and that the receiver plays the strategy in (1). If sender $i \in \{1,2\}$ has type $x_i = 0$, he prefers to reveal w. If sender 1 has type $x_1 < 0$, he prefers to jam w if and only if $w \in (y_d - 2|x_1|, y_d)$, and if sender 2 has type $x_2 > 0$, he prefers to jam w if and only if $w \in (y_d, y_d + 2|x_2|)$.*

Proof. If $x_i = 0$, sender i prefers $y = y_w$ and is (i) indifferent between sending the truthful message and sending any other message if his opponent jams, and (ii) prefers to send the truthful message if his opponent sends the truthful message. Given alternatives y_d and y_w, i with type x_i prefers y_d iff $u(w, x_i, y_w) < u(w, x_i, y_d)$ iff $-|x_i| < -|x_i - (w - y_d)|$. For $x_i < 0$, $-|x_i| < -|x_i - (w - y_d)|$ iff $y_d > w$ and $w > y_d - 2|x_i|$. For $x_i > 0$, $-|x_i| < -|x_i - (w - y_d)|$ iff $y_d < w$ and $w < y_d + 2|x_i|$. ■

Lemma 4. *Assume w and $m_J(w)$ satisfy equation (2), and that the receiver plays the strategy in (1). If $w > y_d$, sender 1 prefers to send $m_1 = w$ and sender 2 prefers to send $m_2 = \left\{ \begin{array}{c} w \\ m_J(w) \end{array} \right\}$ if $x_2 \left\{ \begin{array}{c} < \\ > \end{array} \right\} \frac{1}{2}(m_1 - y_d)$. If $w < y_d$, sender 2 prefers to send $m_2 = w$ and sender 1 prefers to send $m_1 = \left\{ \begin{array}{c} w \\ m_J(w) \end{array} \right\}$ if $x_1 \left\{ \begin{array}{c} > \\ < \end{array} \right\} -\frac{1}{2}(y_d - m_2)$.*

Proof. If $x_i < 0$ and $y_d > w$ then, then $w > y_d - 2|x_i|$ iff $x_i < -\frac{y_d - w}{2}$. If $x_i > 0$ and $y_d < w$, then $w < y_d + 2|x_i|$ iff $\frac{w - y_d}{2} < x_i$. ■

Lemma 5. If senders use the strategies in (1) and $m_1 = m_J(m_2)$, the receiver's beliefs are described by

$$h(w|m_1, m_2) = \left\{ \begin{array}{c} \dfrac{1 - F_2\left(\frac{1}{2}(m_1 - y_d)\right)}{1 - F_2\left(\frac{1}{2}(m_1 - y_d)\right) + F_1\left(-\frac{1}{2}(y_d - m_2)\right)} \\ 1 - h(m_1|m_1, m_2) \\ 0 \end{array} \right\} \text{ if } w \left\{ \begin{array}{l} = m_1. \\ = m_2. \\ \notin \{m_1, m_2\}. \end{array} \right. \quad (3)$$

Proof. If the receiver observes m_1 and $m_2 = m_J(m_1)$, then either sender 2 jammed the truthful message sent by sender 1, so $w = m_1$ and $x_2 > \frac{1}{2}(m_1 - y_d)$, or 1 jammed the truthful message sent by 2, so $w = m_2$ and $x_1 < -\frac{1}{2}(y_d - m_2)$. To apply Bayes' rule, first note that the probability 2 jams $w = m_1$ is $1 - F_2\left(\frac{1}{2}(m_1 - y_d)\right)$, the probability that $x_2 > \frac{1}{2}(m_1 - y_d)$. Similarly, the probability 1 jams $w = m_2$ is $F_1\left(-\frac{1}{2}(y_d - m_2)\right)$. Since w has an atomless, increasing distribution, the events that $w = m_1$ and $w = m_2$ each have prior probability 0. Thus, the posterior probability that $w = m_1$ is given by (3). ■

References

Battaglini, Marco. 2002. "Multiple Referrals and Multidimensional Cheap Talk." *Econometrica* 70 (4): 1379–1401.

Krishna, Vijay, and John Moran. 2001. "A Model of Expertise." *Quarterly Journal of Economics* 116 (2): 747–75.